Optimal tolerancing in mechanical design using polyhedral computation tools

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1 Introduction

Mechanical engineering is a field in which mechanisms (assemblies of parts) are designed and manufactured. This mechanism is born from a customer need and this last specifies technical functions which the final product will have to fulfil. A specification textbook is written. The concern of the designer is then to translate these functions through technological choices. A universal language is essential to define the product characteristics and must be common to each sector of its development (design, manufacture and control). This language is called dimensioning and tolerancing. Each part is represented by its nominal geometry (which is represented in CAD software) based on perfect dimensions. Moreover, these parts are manufactured, so they have defects. It is thus necessary to define acceptable limits in term of form, dimensions and position of functional features. Tolerancing is an important operation because of this one will depend the correct operation of the mechanism but also its cost (the manufacturing cost increases with the precision of tolerances values); one can then be astonished by the absence of tolerancing assistance modules in CAD software. A research team of LMécA (Laboratoire de Mécanique Appliquée) works on a model intended to be integrated in CAD system. This model is called model of the clearance domains and deviation domains [1]. These domains are polytopes built in the six dimensional Euclidean space $\mathbb{R}^6$. For tolerancing analysis, various geometrical operations (Minkowski sums, intersections...) on these polytopes are used (according to the mechanism configuration : single loop, parallel loops or open loop).

In order to perform geometrical operations on polytopes in higher dimensions, we have tested the polyhedral computation library CDDLIB [2]. CDDLIB provides two fundamental operations on convex polytopes, the vertex enumeration and the facet enumeration. More precisely, the vertex enumeration is to obtain the minimal V-representation of an H-polyhedron, and the facet enumeration is the converse. The library can be used with both floating-point arithmetic and infinite precision rational (GMP) arithmetic. Even for simple examples, the floating point computation was seen unstable. We could successfully use the GMP version of CDDLIB for several models of the clearance domains and deviation domains.

2 Clearance Domain

The first function to fulfill for a mechanism is the assemblebility of its constituting parts. These parts are connected the ones to the others by means of joints. It is necessary to consider these joints in the model by defining clearance domain associated to each joint. A joint is constituted

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of two distinct parts and a reference frame is attached to each part. Clearance domain of the joint is a 6D polytopes (3 translations and 3 rotations) whose vertices correspond to maximum displacements of a reference frame compared to the other.

Example (Clearance domain of a cylindrical joint).
For this joint, translation and rotation along the axis x are infinite ($Tx = Rx = \infty$). There are also small displacements due to clearance ($J = D - d$) of the part (1) compared to (0): Ty, Tz, Ry and Rz

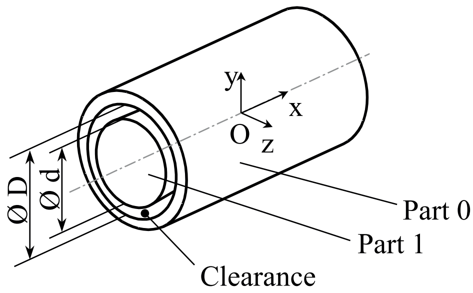

Figure 1: Cylindrical joint

By considering small displacements torsors [3] of the joint, it is possible to write a linear set of inequalities which leads on contact conditions. Those inequalities define the clearance domain of the joint A between the parts (1) and (0). The domain noted $\{J_{0A1}\}$ is unbounded in Tx and Rx directions, a 3D representation is shown in Figure 2.

This representation is a 3D cut of a 6D polytope with $Tx = Rx = Ty = 0$.

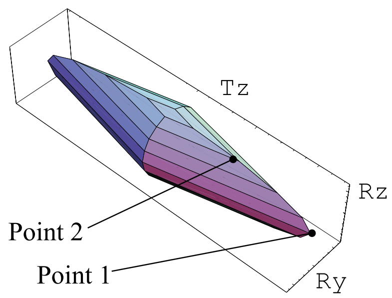

Figure 2: 3D representation of $\{J_{0A1}\}$

Point 1 means: when there is no rotation of part (1) compared to (0) in the joint (nominal position, see fig.1), maximal translation of (1) along $\overrightarrow{z}$ axis is equal to $J/2$. For the point 2, if there is a small rotation around $\overrightarrow{z}$ between the two parts, the translation will be less than $J/2$. All the joint configurations are considered through the 6D domain.

3 Deviation Domain

Tolerancing and so specifications translate designer requirements to define acceptable defect limits of functional features (surface, axis...) with standard language (see Figure 3).

The specification proposed in Figure 3 means that toleranced surface must be positioned (symbol) at a distant a with a tolerance value of t compared to the reference A. The control of such a requirement consists in checking if the manufactured surface is gap between two virtual plans (dotted zone in Figure 3) which are parallel to the reference A and distant of a t value one from

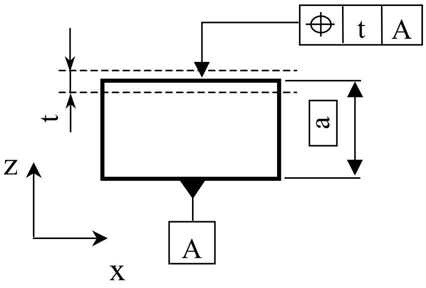

Figure 3: Specification example

the other. In the 6D configuration space (3 translations and 3 rotations), displacements of characteristic points Pi of the toleranced surface are expressed through a set of linear inequalities: $-t/2 \leq \overrightarrow{\delta Pi}.\overrightarrow{z} \leq t/2$. The 6-polytope built with this set of inequalities is the deviation domain $\{E\}$ (see Figure 4) associated to the position specification. It represents the maximum defects (displacements and angular position) of the toleranced surface.

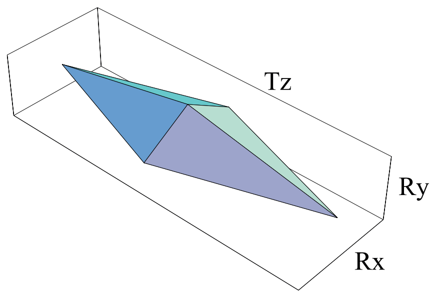

Figure 4: Deviation domain $\{E\}$ of the position specification

4 Application to a Single Loop Mechanism

The mechanism is composed of three parts (see Figure 5):

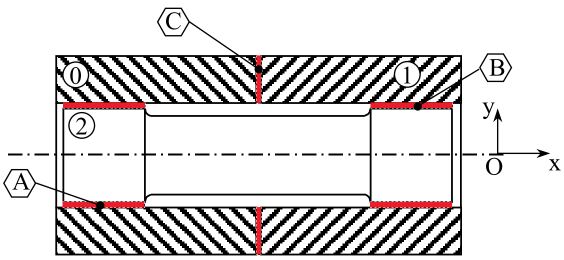

Figure 5: Assembly drawing

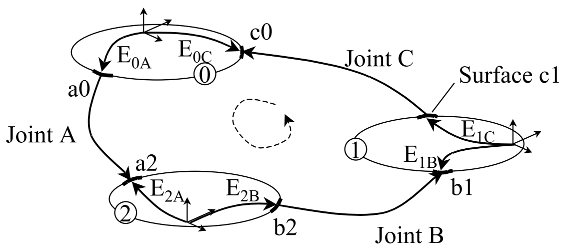

Figure 6: Mechanism diagram

The three joints (A, B and C) can be translated by three clearance domains built in the same point O. Surfaces of each part constituting joint are functional so supposed to be toleranced (engineering drawing is not showed here) and the deviation domains are built in point O. Mechanism theory [4] and configuration of mechanism diagram allow to write the following equation:

$$E_{0C} + J_{0C1} - E_{1C} + E_{1B} + J_{1B2} - E_{2B} + E_{2A} + J_{2A0} - E_{0A} = 0 \quad (4.1)$$

Chosen tolerancing must verify assemblebility and interchangeability of any part belonging to a batch. Fixing $\{J\} = \{J_{0C1}\} + \{J_{1B2}\} + \{J_{2A0}\}$ and $\{E\} = \{E_{0C}\} - \{E_{1C}\} + \{E_{1B}\} - \{E_{2B}\} + \{E_{2A}\} - \{E_{0A}\}$, considering equation (4.1), the two conditions above are satisfied if $\{E\} \subseteq \{J\}$. In other words, more the functional features defects are important, more clearance into mechanism joint will be necessary to correct those defects. From this condition, the procedure of tolerancing analysis is

- Building each clearance domains (6-polytopes) associated to each joint,
- Making Minkowski sum of these domains $\rightarrow \{J\}$,
- Building deviation domains associated to each specification,
- Making Minkowski sum of these domains $\rightarrow \{E\}$,
- Checking the inclusion of $\{E\}$ into $\{J\}$.

Tolerancing is optimal when $\{J\} = \{E\}$.

5 Conclusion

In this model, polytopes help the designer to validate his tolerancing choices (tolerancing analysis): checking mechanism assemblebility, respect of functional requirements. 3D cuts of graphic representations of domains can also inform the designer on his qualitative and quantitative tolerancing choices (tolerancing synthesis).

While existing codes for polyhedral computation turned out to be useful for analyzing some models of the clearance domains and deviation domains, we need further developments of polyhedral computation codes. In particular, there is no efficient codes to compute the Minkowski addition [5] of convex polytopes. In fact, this motivates one of the authors to design a new efficient algorithm for this problem which is highly parallelizable and easy to implement, see [6]. Implementing this algorithm and applying to our models, in particular for tolerancing analysis, is one of our future research projects.

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