Online escape paths
joint work with David Kübel

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Lost in a forest: Ultimate escape path
Lost in a forest: Ultimate escape path

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Lost in a forest: Ultimate escape path
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Lost in a forest: Ultimate escape path

Adversary rotates/translation

Diagram showing a forest with a marked path and an adversary's possible movements.
Lost in a forest: Ultimate escape path

Adversary rotates/translations
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Adversary rotates/boatenlates
Lost in a forest: Ultimate escape path

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Adversary rotates/translations

Zig-Zag path is optimal

$$\sqrt{\frac{27}{28}} \approx 0.98199$$
Lost in a forest: Ultimate escape path

- Bellman (1956): Lost in a forest
- Besicovitch (1965): Zig-Zag-Path
- Coulton/Movshovich (2006): Optimality
- Very few examples Strip/Zalgaller

\[ \sqrt{\frac{27}{28}} \approx 0.98199 \]
Lost in a forest: Certificate path

- Ultimate escape path only known for some convex shapes
- I. Adversary only rotates! II. Simple escape path!

![Diagram of a triangle with labels A, B, C and an adversary point s.](image-url)
Lost in a forest: Certificate path

- Ultimate escape path only known for some convex shapes
- I. Adversary only rotates! II. Simple escape path!

![Diagram of a triangle with labels A, B, C and points s, 1]
Lost in a forest: Certificate path

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Adversary rotates!

A

1

s

1

1

B

C

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Online escape paths
Lost in a forest: Certificate path

- Ultimate escape path only known for some convex shapes
- I. Adversary only rotates! II. Simple escape path!

Adversary rotates!

![Diagram of a triangle with labeled vertices A, B, and C, and a point s. The edges are labeled with 1.]
Lost in a forest: Certificate path

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Adversary rotates!
Lost in a forest: Certificate path

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- I. Adversary only rotates! II. Simple escape path!

![Diagram of a triangle with labeled vertices A, B, C and a path marked with an 'x'.]
Lost in a forest: Certificate path

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Segment plus Circular Arc
Lost in a forest: Certificate path

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- I. Adversary only rotates! II. Simple escape path!

Adversary rotates!

Segment plus Circular Arc

Adversary rotates!
Lost in a forest: Certificate path

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![Diagram showing a triangle with points A, B, and C, and a path from A to C via B with labels and angles.](image)

- Segment plus Circular Arc
- Adversary rotates!

Shortest segment/arc combination not covered by rotation!

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Lost in a forest: Certificate path

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Segment plus Circular Arc

Adversary rotates!

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Segment plus Circular Arc

\[ A \rightarrow s \rightarrow B \]

Adversary rotates!

\[ s \rightarrow C \]

\[ x \cdot \alpha \]
Lost in a forest: Certificate path

- Ultimate escape path only known for some convex shapes
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\[ \text{Segment plus Circular Arc} \]

Adversary rotates!

Segment plus Circular Arc
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Adversary rotates!

Segment plus Circular Arc

Shortest segment/arc combination not covered by rotation!
Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information

- Polynomial time
- Non-convex
- Star-shaped
Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information!

Triangle

Zig-Zag path

Certificate

Better candidates!
Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information!

- Triangle
- Zig-Zag path
- Certificate
- Better candidates!
Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information!

Triangle
Zig-Zag path
Certificate
Better candidates!
Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information!

- Triangle
- Zig-Zag path
- Certificate
- Better candidates!
Advantage: Certificate vs. Ultimate

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- Triangle
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Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information!

- Triangle
- Zig-Zag path
- Certificate
- Better candidates!
Advantage: Certificate vs. Ultimate

- Easy to compute, more general environments
- Beats ultimate path, more information!

- Strip
- Zalgaller path
- Certificate
- Better candidates!
Certificate: Extreme Cases!

- Few small distances: $\approx x(1 + 2\pi)$
- All distances roughly the same: $\approx x(1 + 0)$
- Certificate $\approx x(1 + \alpha_x)$ with $\alpha_x \in [0, 2\pi]$!
Another turn of the screw: Online!

- NO sight, NO knowledge of shape AND position!
- Online escape path! Unknown certificate! Spiral strategy!
- Logarithmic spiral vs. certificates! Cover unknown certificate!
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Online escape paths
Online spiral path against arbitrary certificate

- Online spiral escape path with eccentricity $\beta$!
- Detour against arbitrary certificate: Competitive ratio!

Detour: 2.18  Eccentricity: $\beta = 1.29$
Online spiral path against arbitrary certificate

- Online spiral escape path with eccentricity $\beta$!
- Detour against arbitrary certificate: Competitive ratio!

Detour: $3.318764\ldots$

Eccentricity: $\beta = 1.26471\ldots$
Online spiral path against arbitrary certificate

The same worst-case ratio for both extremes! Analytically!

**Theorem I:** There is a spiral strategy that escape from *any unknown environment* (kernel/star-shaped) and has detour at most $3.318764\ldots$ against the corresponding certificate path for the known shape.
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[
\sum_{k=1}^{n-1} \sqrt{x_{2i} - 2 \cos\left(\frac{2\pi}{n}\right)x_{2i}x_{i+1}} + \sum_{k=1}^{n-1} \sqrt{x_{2i+1} - 2 \cos\left(\frac{2\pi}{n}\right)x_{2i+1}x_{k}}
\]

for all \(n\) and \(k\).

Gives at least \(D = 6\). 6.2521...

for the sum of Detours!

Detour at least \(D/2 \geq 3.313126\)!

\(n\) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[
\text{Detour at least } D/2 \geq 3.313126!
\]
\[
\text{n goes to infinity!}
\]
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

Lower bound: Minimize
$$\sum_{k=1}^{n-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right) x_i x_{i+1}} + \sum_{k=1}^{n-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right) x_i x_{i+1}} + x_{i+n-1}(1 + \frac{2\pi}{n})$$

for all $n$ and $k$.

Gives at least $D = 6.2521\ldots$ for the sum of Detours!

Detour at least $D/2 \geq 3.313126!$

$n$ goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[ \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} x_i \right) x_{i+1}} + \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} x_i \right) x_{i+1}} + x_{2k} \left(1 + 2\pi \right) + \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} x_i \right) x_{i+1}} \]

for all \( n \) and \( k \).

Gives at least \( D = \frac{6}{2} = 3.13126 \).

Detour at least \( D/2 \geq 3.313126! \)

\( n \) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[ \text{Detour at least } D/2 \geq 3.313126! \]
\[ n \text{ goes to infinity!} \]
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[ \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos(2\pi/n) x_{2i+1} + x_{2i+1}} \]

for all \( n \) and \( k \).

Gives at least \( D = 6 \).

\[ \sum \frac{D}{2} \geq 3.313126! \]

\( n \) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[
\text{Lower bound: Minimize } \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi n}{x_i} \right) x_i x_{i+1}} + \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi n}{x_i} \right) x_i x_{i+1}} + x_{2k} (1 + 2 \pi n) + \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi n}{x_i} \right) x_i x_{i+1}} \right) \quad \text{for all } n \text{ and } k.
\]

Gives at least \( D = 6 \).62521... for the sum of Detours!

Detour at least \( D/2 \geq 3.313126! \)

\( n \) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[
\sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} \right) x_{2i} x_{2i+1}} + \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} \right) x_{2i} x_{2i+1}} + x_{2i} x_{k-n+1} (1 + 2\pi n) + \sum_{i=1}^{k-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} \right) x_{2i} x_{2i+1}} + x_{2i} x_{k-n+1} (1 + 2\pi n)
\]

for all \( n \) and \( k \).

Gives at least \( D = 6 \).

\[n\] goes to infinity!

Detour at least \( D/2 \geq 3.313126!\)
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[
\text{Detour at least } D/2 \geq 3.313126!
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n \text{ goes to infinity!}
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Lower bound: Minimize
\[
\sum_{k=1}^{n-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} \right) x_{2i+1}} + \sum_{k=1}^{n-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} \right) x_{2i+1}} + x_{2i+1} x_{k} (1+2\pi n) + \sum_{k=1}^{n-1} \sqrt{x_{2i} - 2 \cos \left( \frac{2\pi}{n} \right) x_{2i+1}} + x_{2i+1} x_{k} - n+1 (1+2\pi n)
\]

for all \(n\) and \(k\).

Gives at least \(D = 6.62521\)...

for the sum of Detours!

Detour at least \(D/2 \geq 3.313126\)!

\(n\) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

Detour at least \( D/2 \geq 3.313126! \)

\( n \) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

Detour at least $D/2 \geq 3.313126$!

$n$ goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
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Detour at least $D/2 \geq 3.313126$!

$n$ goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

\[
\sum_{k=1}^{n-1} \sqrt{x_i - 2 \cos(\frac{2\pi}{n}) x_{i+1} x_{i+1}} + \sum_{k=1}^{n-1} \sqrt{x_i - 2 \cos(\frac{2\pi}{n}) x_{i+1} x_{i+1} + x_k} (1 + 2\pi n) + \sum_{k=1}^{n-1} \sqrt{x_i - 2 \cos(\frac{2\pi}{n}) x_{i+1} x_{i+1} + x_k - n+1} (1 + 2\pi n)
\]

for all \(n\) and \(k\).

Gives at least \(D = 6\). \(6^{21}\) for the sum of Detours!

Detour at least \(D/2 \geq 3.313126\)!

\(n\) goes to infinity!
Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

Lower bound: Minimize

\[
\sum_{i=1}^{k-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right)x_ix_{i+1} + x_{i+1}^2} + x_k(1 + \frac{2\pi}{n})
\]

\[
\sum_{i=1}^{k-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right)x_ix_{i+1} + x_{i+1}^2} - x_{k-n+1}(1 + 2\pi)
\]

for all \( n \) and \( k \).

Gives at least \( D = 6.62521 \ldots \) for the sum of Detours!

Detour at least \( D/2 \geq 3.313126! \)

\( n \) goes to infinity!
Adversary rotates/translates

Detour: 3.318764...
Eccentricity: \( \beta = 1.26471... \)
**Theorem:** Escape from an unknown environment against the best certificate can be done within a competitive ratio of $3.318764 \ldots$ and this is (almost $\approx 0.005$) tight.