Inspecting a Set of Strips Optimally

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Abstract. We consider a set of axis-parallel nonintersecting strips in the plane. An observer starts to the left of all strips and ends to the right, thus visiting all strips in the given order. A strip is *inspected* as long as the observer is inside the strip. How should the observer move to inspect all strips? We use the path length outside a strip as a quality measure which should be minimized. Therefore, we would like to find a directed path that minimizes the maximal measure among all strips. We present an optimal algorithm designed according to the structural properties of the optimal solution.

Keywords: Computational geometry, motion planning, watchman routes, optimal inspection path, optimal algorithm

1 Introduction

In the last decades, routes from which an agent can see every point in a given environment have drawn a lot of attention. For example, the optimal offline exploration path in a simple polygon (the *shortest watchman route*) was first considered by Chin and Ntafos [4] for the special case of orthogonal polygons. Some work has been done on shortest watchman routes, until Dror et al. [7] presented an algorithm for the more general problem of visiting a sequence of intersecting polygons under the presence of fences. Similar problems include the shortest watchman *path* with different start and end points [3] or routes with additional constraints such as zookeeper routes [5]. The corresponding online task was considered, for example, by Deng et al. [6] for orthogonal simple polygons, by Hoffmann et al. [8, 9] for general simple polygons, and by Icking et al. [10] for grid polygons. See also the surveys by Mitchell [11] or Icking et al. [9].

Usually, the objective is to find a short path or route (i.e., a closed path); either the shortest possible route (the optimum) or an approximation. In this paper, we focus on another criterion for routes: We want to minimize the time[3] in which a certain area of the environment is *not* seen. Imagine a guard in an art gallery whose objective is to be as vigilant as possible and to minimize the time an object is unguarded. Thus, for a given *inspection route* the route's performance wrt. a single object is given by the maximal time interval where

[3] We assume that the agent travels with constant speed; thus, we use *time* and *path length* synonymously.

the object is unguarded. The task is to find a path that minimizes the maximal unguarded time interval among all objects.

Mark Overmars [12] introduced this problem by posing the question whether the shortest watchman route inside a simple polygon is the best inspection route. In this setting the set of objects is given by the set of *all* points inside the polygon. The shortest watchman route *inspects* some of these points only in a single moment. Therefore the conjecture is that the performance of the optimal inspection route for the polygon is given by the length of the shortest watchman route. This question is still open. In this paper, we restrict ourselves to a simple type of environments—parallel strips in the plane. More complicated environments are the subject of ongoing research. We present an optimal algorithm that solves the problem for L_1- and L_2-metric and gives some insight into the structural property of optimal inspection paths.

The paper is organized as follows. In Section 2, we present notational conventions and define an objective function which has to be minimized. Then, in Section 3 we first prove some structural properties of an optimal solution for the Euclidean case. At the end we present an efficient algorithm. The ideas can be adapted to the L_1-case which is mentioned in Section 4. Finally, we summarize the results and discuss future work, see Section 5.

2 Preliminaries

Let $\{S_1, \ldots, S_n\}$ be a set of nonintersecting vertical strips, $S = (s_x, s_y)$ be a start point to the left of all strips, and $T = (t_x, t_y)$ be an end point to the right of all strips. W.l.o.g. we can assume that S is below T (i.e., $s_y \leq t_y$). Strip S_i has width w_i.

An inspection path, P, from S to T visits the strips successively from left to right, see Fig. 1. For a given path P let P_i denote the part of P within strip S_i. Let $|P_i|$ denote the corresponding path length, and last(P) the last segment of P (i.e., from S_n to T).

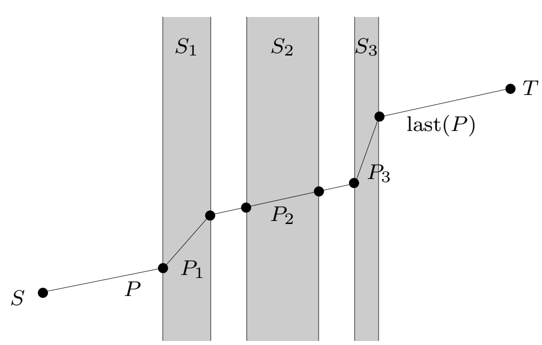

Fig. 1. Visiting three strips in a given order.

While P visits S_i, the strip is entirely visible. The performance of P for a single strip S_i therefore is given by $\mathrm{Perf}(P, P_i) := |P| - |P_i|$. The performance of the path P for all strips is given by the worst performance achieved for a single strip. That is $\mathrm{Perf}(P) := \max_i \mathrm{Perf}(P, P_i)$. Finally, the task is to find among all inspection paths the path that gives the best performance for the given situation; that is, an inspection path with minimal performance:

$$\mathrm{Perf} := \min_P \max_i \mathrm{Perf}(P, P_i) \; .$$

This problem belongs to the class of LP-type problems [14], but the basis could have size n as we will see later. Therefore, we solve the problem directly. It may also be seen as a *Time and Space* Problem (see, e.g., [2, 1]).

3 The Euclidean Case

In this section, we first collect some properties of the optimal solution and then design an efficient algorithm.

3.1 Structural Properties

We can assume that the optimal inspection path is a polygonal chain with straight line segments inside and between the strips. There can be no arcs or kinks inside the strips or outside the strips (in the *free space*). The inspection path enters a strip and leaves a strip and the straight line between these points does not influence the performance of the corresponding strip but optimizes the length of the corresponding subpath. Analogously, between two strips the path leaves a strip and enters another strip and the straight line between these point optimizes the length of the corresponding subpath.

Let us further assume that we have an inspection path as depicted in Fig. 2(i). The first simple observation is that we can rearrange the set of strips in any nonintersecting order and combine the elements of the given path adequately by shifting the segments horizontally without changing the path length; see Fig. 2(ii).

Now, it is easy to see that an optimal solution has the same slope between two successive strips. We simply rearrange the strips such that they stick together and start from the X-coordinate of the start point. The last part of the solution should have no kinks as mentioned earlier.

Lemma 1. *The optimal solution is a polygonal chain without kinks between the strips or inside the strips. The path has the same slope between all strips.*

In the following, we assume that the strips are ordered by widths $w_1 \leq w_2 \leq \cdots \leq w_n$, starting at the X-coordinate, s_x, of the start point and lie side by side (i.e., without overlaps or gaps), see Fig. 2(ii). For $t_y = s_y$ the optimal path is simply the horizontal connection between S and T. Thus, we assume $t_y > s_y$ the following.

Now, we show some structural properties of an optimal solution. The optimal solution visits some strips with the same value $d = |P_i|$ until it finally moves directly to the end point, see Fig. 3 for an example.

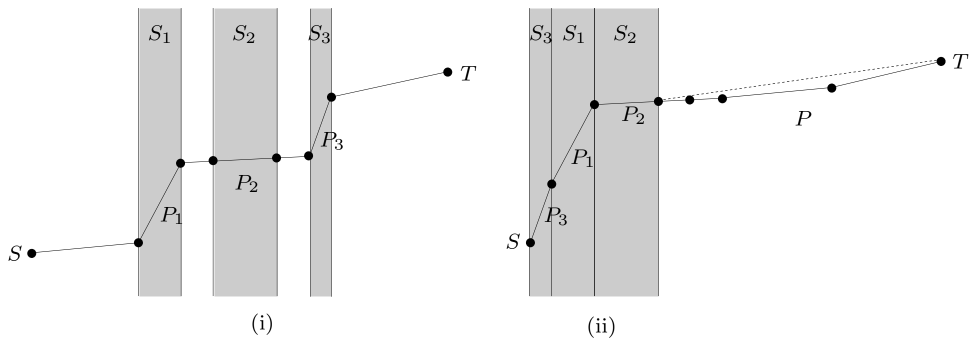

Fig. 2. Rearranging strips and path yields the same objective value. Thus, the optimal solution has the same slope between all strips and we can assume that the strips are ordered by widths.

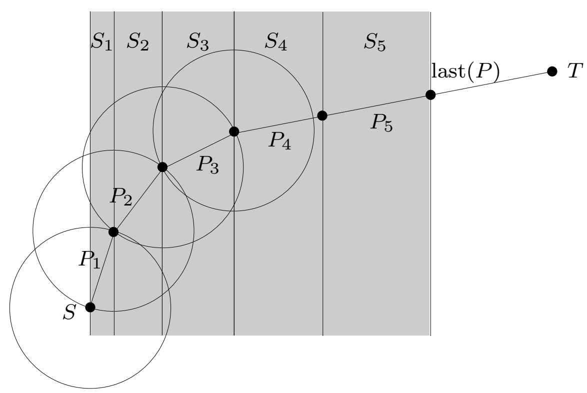

Fig. 3. The structure of an optimal solution: The first three strips are visited with the same value $|P_i| = d$, every other strip with $|P_i| > d$.

Lemma 2. *In a setting as described above, the optimal path P visits the first $k \leq n$ strips with the same distance d and then moves directly to the end point. That is, for $i = 1, \ldots, k$ we have $|P_i| = d$ and for $i = k+1, \ldots, n$ we have $|P_i| > d$. The path is monotonically increasing and convex with respect to the segment ST.*

Proof. Let P denote an optimal path for n strips. First, we show that the path is monotonically increasing. There is at least one segment, P_i, with positive slope, because $t_y > s_y$. Let us assume—for contradiction—that there is also a segment, P_j, with negative slope. We rearrange the strips and the path such that P_i immediately succeeds P_j, see Fig. 4(i). Now, we can move the common point of P_j and P_i upwards. Both segments decrease and the performance of P improves. Thus, there is no segment with negative slope and P is monotonically increasing.

The performance of P is given by $|P_k| := \min_j |P_j|$. Let k be the greatest index such that P_k is responsible for the performance. By contradiction, we

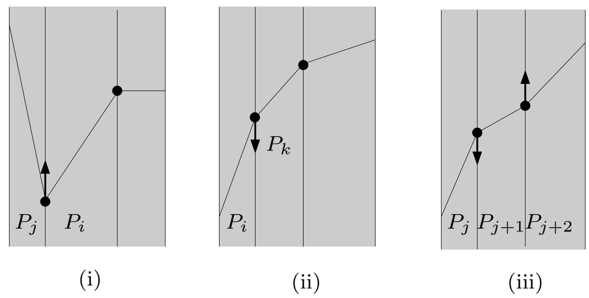

Fig. 4. Global optimization by local changes: The connection point can be moved (i) upwards, (ii) downwards or (iii) upwards or downwards. In any case the solution can be improved by local changes.

show that for $i < k$ there is no P_i with $|P_i| > |P_k|$. So let us assume that $|P_i| > |P_k|$ holds for $i < k$. We can again rearrange the strips in such a way that P_k immediately succeeds P_i. From $w_i \leq w_k$ we conclude that the path P_iP_k makes a right turn. Because $|P_i| > |P_k|$ holds, we can globally optimize the solution by moving the connection point downwards, see Fig. 4(ii). Although $|P| - |P_i|$ increases, the total path length decreases. Thus, $|P_i| = |P_k|$ for $i = 1, \ldots, k-1$. For $i = k+1, \ldots, n$ we have $|P_i| > |P_k|$ by assumption.

Now, we show that there is no kink in the path $P_{k+1}P_{k+2}\cdots P_n$. As $|P_j| > |P_k|$ for $j > k$ we can globally optimize the solution by moving a kink point downwards or upwards, see Fig. 4(iii). The path length decreases. Therefore, $P_{k+1}P_{k+2}\cdots P_n$ is a straight line segment.

Altogether, for $i = k+1, \ldots, n$ we have $|P_i| > d$. For $i = 1, \ldots, k$ we have $|P_i| = d$ and the part $P_{k+1}P_{k+2}\cdots P_n\text{last}(P)$ is a straight line segment. Path P is monotonically increasing. The first part of P makes only right turns as already seen. The last part is a line segment. The concatenation of P_k and $P_{k+1}P_{k+2}\cdots P_n$ also makes a right turn; otherwise, we can again improve the performance. Because $|P_{k+1}| > |P_k|$ we can move the connection point upwards, which decreases the path length and improves the performance of strip S_k and S_{k+1}. Altogether, P is convex with respect to ST. □

One might think that with the result of Lemma 2 there is an easy way to find a solution by application of Snell's law, which describes how light bends when traveling from one medium to the next. In the formulas below an application or extension of Snell's law seems to be difficult to achieve.

In the following, we show that we can compute the optimal solution incrementally; that is, we successively add new strips and consider the corresponding optimal solutions.

Let $S_1, S_2, \ldots, S_n$ be a set of strips and let S and T be fixed. Let P^i denote the optimal solution for the first i strips. For increasing i the parameter k of Lemma 2 is strictly increasing until it remains fixed:

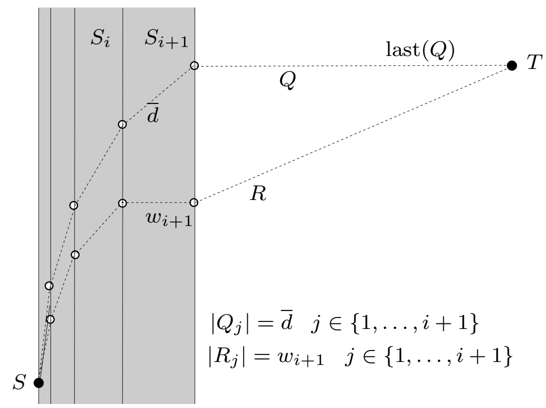

Fig. 5. An optimal solution for $k = i + 1$ strips can be found between the extremes R and Q. $|R_j| = w_{i+1}$ holds and $\overline{d}$ fulfills $|Q_j| = \overline{d}$ with horizontal last(Q).

Lemma 3. *For P^i either the index k (see Lemma 2) is equal to i or P^i is already given by P^{i-1}. If P^i is identical to P^{i-1}, also P^j is identical to P^{i-1} for $j = i+1, \ldots, n$.*

We postpone the complete proof of Lemma 3 and first show how to adapt a solution P^i to a solution P^{i+1}. Let us assume that we have a solution P^i and that k in the sense of Lemma 2 is equal to i. That is, the first i strips are visited with the same distance d. If $|P^i_{i+1}| < d$ holds, the solution P^i is not optimal for $i+1$ strips. Therefore we want to adapt P^i. Lemma 3 states that it migth be useful to search for a solution with identical path length in $k = i+1$ strips.

We now show that this solution can be computed efficiently. The task is to compute the path P^{i+1} between two extreme solutions, R and Q, as follows: Let R be the path with path lengths $|R_j| = w_{i+1}$ for $j = 1, \ldots, i+1$ and let Q be the path with $|Q_j| = \overline{d}$ for $j = 1, \ldots, i+1$, where the last segment, last(Q), is horizontal. Starting from $d = w_{i+1}$, let $P^{i+1}(d)$ denote the unique path that starts with $|P^{i+1}_j| = d$ for $j = 1, \ldots, i+1$ and ends with a straight line segment, last$(P^{i+1}(d))$. The path R always exists, we can construct it by starting from the first strip. The path Q exists only if the path R does not exceed the Y-coordinate t_y of T. In this case, the strip S_{i+1} will have no influence on the optimal solution in the sense of Lemma 2, an optimal solution will directly pass through S_{i+1} and the following strips.

So let us assume that R and Q exist and that we would like to compute the performance of $P^{i+1}(d)$ starting at $d = w_{i+1}$. The performance of $P^{i+1}(d)$ is given by the function

$$f_{i+1}(d) := d \cdot i + |\text{last}(P^{i+1}(d))| \,.$$

Note that we can express $|\text{last}(P^j(d))|$ in terms of d: For convenience, let y_j be the vertical height of $P^{i+1}_j(d)$ (i.e., $d^2 = w_j^2 + y_j^2$), and X be the horizontal distance from the last strip to T. With $T = (t_x, t_y)$ and $S = (0, 0)$ we have

$X := t_x - \sum_{j=1}^{i+1} w_j$. Now, we have $|\text{last}(P^{i+1}(d))| = \sqrt{X^2 + \left(t_y - \sum_{j=1}^{i+1} y_j\right)^2}$. With $y_j := \sqrt{d^2 - w_j^2}$ we get

$$f_{i+1}(d) = d \cdot i + \sqrt{X^2 + \left(t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}\right)^2}. \tag{1}$$

By simple analysis, we can show that $f_{i+1}(d)$ has a unique minimum in d while increasing d from w_j until $\text{last}(P^{i+1}(d))$ gets horizontal:

Lemma 4. *The function $f_{i+1}(d)$ (Eq. 1) has exactly one minimum for $d \in [w_{i+1}, \bar{d}]$, where $\bar{d}$ is the solution of $t_y - \sum_{j=1}^{i+1} \sqrt{\bar{d}^2 - w_j^2} = 0$ (i.e., the last segment is horizontal).*

Proof. Let us consider the first derivative of $f_{i+1}(d)$ in d, which is given by

$$f'_{i+1}(d) = i - \frac{t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}}{\sqrt{X^2 + \left(t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}\right)^2}} \cdot \sum_{j=1}^{i+1} \frac{d}{\sqrt{d^2 - w_j^2}}, \tag{2}$$

where $X := t_x - \sum_{j=1}^{i+1} w_j$. The first summand, i, of Eq. 2 is constant. The second summand of Eq. 2 is given by

$$h_{i+1}(d) := \frac{t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}}{\sqrt{X^2 + \left(t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}\right)^2}} \cdot \sum_{j=1}^{i+1} \frac{d}{\sqrt{d^2 - w_j^2}}.$$

If $h_{i+1}(d)$ is strictly monotone in d, there will be at most one solution for $f'_{i+1}(d) = 0$. Note that $h_{i+1}(d)$ is always positive. Let us consider the derivative of $h_{i+1}(d)$ which is $g'_{i+1}(d) l_{i+1}(d) + g_{i+1}(d) l'_{i+1}(d)$ for

$$g_{i+1}(d) = \frac{t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}}{\sqrt{X^2 + \left(t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}\right)^2}} \quad \text{and} \quad l_{i+1}(d) := \sum_{j=1}^{i+1} \frac{d}{\sqrt{d^2 - w_j^2}}.$$

It is clear that $l_{i+1}(d) > 0$ and $g_{i+1}(d) > 0$ holds until $\text{last}(P^{i+1}(d))$ is horizontal. It remains to show that $g'_{i+1}(d)$ and $l'_{i+1}(d)$ both have negative sign. This means that $h_{i+1}(d)$ is strictly decreasing and $i - h_{i+1}(d)$ changes from positive to negative only once. Thus, $f_{i+1}(d)$ has a unique minimum.

By simple derivation we have

$$l'_{i+1}(d) = -\sum_{j=1}^{i+1} \frac{w_j^2}{(d^2 - w_j^2)\sqrt{d^2 - w_j^2}} \quad \text{and}$$

$$g'_{i+1}(d) = -\sum_{j=1}^{i+1} \frac{d}{\sqrt{d^2 - w_j^2}} \cdot \frac{X^2}{\left(X^2 + \left(t_y - \sum_{j=1}^{i+1} \sqrt{d^2 - w_j^2}\right)^2\right)^{\frac{3}{2}}}$$

which both have negative sign in the given interval for d. Altogether, the statement follows. $\square$

Now, it is easy to successively compute the minimum of $f_{i+1}(d)$ for $i = 0, \ldots, n-1$. For example, we can apply efficient numerical methods for getting a solution of $f'_{i+1}(d) = 0$, especially because of the strictly decreasing behaviour of f'_{i+1}. In the following, we assume that we can compute this minimum in time proportional to the number of terms of the given functions, i.e., the minimum of $f_{i+1}(d)$ is computed in $O(i)$. This assumption is well justified. We can choose an appropriate starting interval and numerical methods will achieve a good convergence rate, for details see Schwarz [13].

Note that Eq. 2 contains the sum of cosines of the bending angles in the strips times the cosine of the bending angle in the last segment. The parameter d has to be adjusted and it changes all angles simultaneously. That is, there is a global criterion involved and, thus, it seems hard to find a simple ratio that resembles the refractive index in Snell's law.

Using Lemma 4 we can now prove Lemma 3:

Proof of Lemma 3. Let us assume that we have a solution P^i computed for i strips and let $|P^i_k| = d$, where k denotes the index in the sense of Lemma 2. We use this solution for the first $i+1$ strips. If $|P^i_{i+1}| \geq d$ holds, the given solution is also optimal for $i+1$ strips because the overall performance remains the same: The last segment, $\text{last}(P^i)$, of P^i is a line segment with positive slope. Further, $w_j \geq w_{j-1}$ holds. Thus, P^i is the overall optimum; that is, we can apply P^i to all n strips and $|P^i_j| \geq d$ holds for $j = i+1, i+2, \ldots, n$.

This means: If we have found a solution P^i with $|P^i_{i+1}| \geq |P^i_k|$ where k denotes the index from Lemma 2, then we are done for all strips.

It remains to show that the index k is strictly increasing until it is finally fixed. From the consideration above we already conclude that there is only one strip, for which k does not increase. Namely, if k does not increase from i to $i+1$ we have $k < i+1$ and $d = |P^{i+1}_k| < |P^{i+1}_j|$ for $j = k+1, \ldots, n$. Thus, P^{i+1} is the overall optimum and k is fixed.

Finally, we show that indeed k can never decrease (i.e., fall back to some $k < i$) if strip S_{i+1} is added. Let us assume from $\ell = 1$ to $\ell = i$ we have always a solution P^ℓ for ℓ strips and $k = \ell$ for every P^ℓ. Let us further assume that for $i+1$ strips the solution P^{i+1} comes along with $k = j < i$. We compare the two solutions P^{i+1} (with $k = j < i$ and $|P^{i+1}_j| =: d_{i+1}$) and P^j (with $k = j$ and $|P^j_j| =: d_j$), see Fig. 6.

As P^j is optimal for j strips but not for $j+1$ strips, we have $|P^j_{j+1}| < d_j$. On the other hand, P^{i+1} is optimal for $i+1$ strips. Thus, $|P^{i+1}_{j+1}| > d_{i+1}$ holds; see Lemma 2.

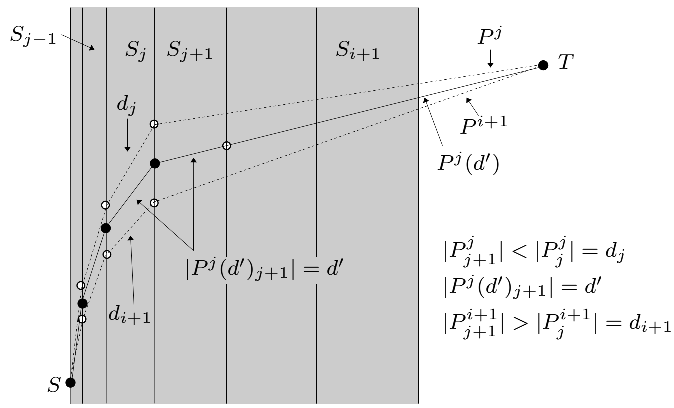

Fig. 6. Between P^{i+1} and P^j there has to be a solution with $P(d)$ which is better than P^{i+1}.

Now for a parameter d consider a monotone path $P^j(d)$ that starts from S, has equal path length d in the first j strips and then moves toward T. While d increases, the slope of the last segment strictly decreases. Therefore, the path length $|P^j(d)_{j+1}|$ is strictly decreasing in d. This means that $d_j > d_{i+1}$ and P^j runs above P^{i+1}. The path $P^j(d)$ changes continuously, therefore in $[d_{i+1}, d_j]$ there has to be a value d' such that $|P^j(d')_{j+1}|$ is equal to d', see Fig. 6. The path $P^j(d')$ runs between P^{i+1} and P^j. Obviously, $d' < P^j(d')_l$ holds for $l = j+1, \ldots, n$.

We show that $P(d')$ is better than P^{i+1}. This is a direct consequence of Lemma 4. The value of $P^j(d)$ strictly decreases from $d = d_{i+1}$ to the unique minimum $d = d_j$ and d' is in between. Altogether, P^{i+1} is not optimal which is a contradiction. □

The result of Lemma 3 now suggests a method for computing the optimal path efficiently. Starting from $j = 1$ we compute an optimal path for the first j strips. Let P^j denote this path. If $|P^j_{j+1}| > |P^j_j|$ holds we are done. Otherwise, we have to compute P^{j+1} for $j+1$ strips and so on.

3.2 Algorithm and its Analysis

Theorem 5. *For a set of n axis-aligned strips the optimal inspection path can be computed in $O(n \log n)$ time and linear space.*

Proof. First, we sort the strips by width which takes $O(n \log n)$ time. Then we apply binary search. That is, in a first step we compute a solution with respect to $j = \lfloor \frac{n}{2} \rfloor$ strips.

We compute the path R with $|R_l| = w_j$ for $l = 1, \ldots, j$. If R does already exceeds the Y-coordinate t_y of T, the optimal path should directly pass through S_j and all successive strips as mentioned above. Therefore we proceed with the interval $[1, j]$ in this case. Otherwise, we compute the best value for $f_j(d)$ starting from $w_j = d$ until the last segment is horizontal, see Lemma 4. Let d_j denote the optimal value for j strips and P^j the optimal path.

Now, we have to determine whether the optimal path visits $i \leq j$ or $i > j$ strips with the same distance d_i. If $d_j > |P^j_{j+1}|$, we have to take into account at least the strip S_{j+1}. Therefore, $i > j$ holds and we proceed recursively with the interval $[j+1, n]$. If $d_j \leq |P^j_{j+1}|$ we proceed with the interval $[1, j]$. Therefore we will find the optimum in $\log n$ steps. Computing R and the minimum of $f_j(d)$ for index j takes $O(j)$ time. □

For a lower bound construction we can simply assume that the input of an algorithm is given by an unsorted set of strips. The X-coordinates of the strip's left boundaries and their widths describe the setting. The solution is given by a polygonal chain from left to right representing the order of the left boundaries. Thus, sorting a set of n elements can be reduced to the given problem.

Theorem 6. *For a set of n axis-aligned unsorted strips the optimal inspection path is computed in $\Theta(n \log n)$ time and $\Theta(n)$ space.*

4 The L_1-Case

Fortunately, if we measure the distance by the L_1 metric the structural properties are equivalent. Computing the optimal path becomes much easier.

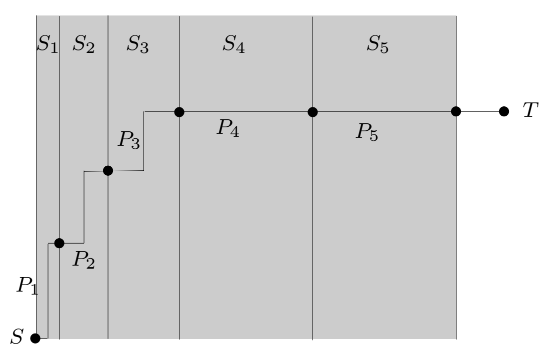

Fig. 7. The structure of an optimal solution in the L_1-case after rearrangement. The first three strips are visited with the same value d, for all other strips $|P_i| > d$ holds. The path is horizontal between strips.

Note that the path segments between the strips have to be horizontal. We have to distribute the vertical distance from S to T among a subset of the strips. Again we sort the strips by their widths and rearrange the scenario. Fig. 7 shows an example of an optimal L_1-path after rearrangement.

Theorem 7. *The optimal L_1-path P visits the first $k \leq n$ strips with the same L_1-distance d and then moves horizontally to the end point T. For $i = 1, \ldots, k$ we* *have* $|P_i| = d$ *and for* $i = k+1, \ldots, n$ *we have* $w_i > d$. *Additionally,* $\sum_{i=1}^{k} |P_i| = t_y$ *holds. If the number of strips, n, increases, the index k increases until it remains fixed. The optimal path can be computed in* $\Theta(n \log n)$ *time and* $\Theta(n)$ *space.*

An algorithm for the L_1 problem is given as follows. First, we sort the strips by their widths. Then starting from $i = 1$ we distribute $t_y + \sum_{j=1}^{i} w_j$ among i strips. For an optimal path, P^i, for i strips we have $|P_j^i| = \frac{1}{i}(t_y + \sum_{j=1}^{i} w_j)$ for $j = 1, \ldots, i$. If $|P_i^i| < w_{i+1}$ this path is also optimal for $i+1$ (and n) strips. For $|P_i^i| > w_{i+1}$ we distribute $t_y + \sum_{j=1}^{i+1} w_j$ among $i+1$ strips; that is, $|P_j^{i+1}| = \frac{1}{i+1}(t_y + \sum_{j=1}^{i+1} w_j)$ for $j = 1, \ldots, i+1$. We can compute the sum of the weights successively. For a single step only a constant number of operations is necessary.

Altogether, if the strips are given by ordered widths, the algorithm runs in $\Theta(n)$ time and space.

5 Conclusion and Future Work

We presented an optimal algorithm that computes the shortest inspection path for a set of axis-aligned strips which has to be visited in a given order.

The performance of a path P for a single strip S_i is given by the time where the strip is not inspected, i.e. $|P| - |P_i|$. The maximum value $|P| - |P_i|$ among all strips gives the performance of the inspection path. In turn, we compute a path P with minimal performance among all paths in optimal time and space. The approach works for L_1- and L_2-metric.

The structural properties of the solution show that a set of strips with increasing widths determines the solution, the remaining strips are of greater widths and they will be simply passed. This shows that the problem is of LP-Type [14]. The set of strips is the set H and $w(G)$ gives the performance of the optimal solution for a subset $G \subseteq H$. Obviously, w is *monotone*, that is, if we add more strips the performance of the solution cannot decrease. On the other hand *monotonicity* holds. If two subsets $F \subseteq G \subseteq H$ have the same performance and adding an additional strip $h \in H$ does not change the performance of F, the strip h can also not change the performance of G if added. Unfortunately, all strips might determine the solution. Thus the basis of the problem is not a single constant and it was worth computing a solution directly.

One might think that a relative performance is more intuitive. That is, for an inspection path P, $\frac{|P|}{|P_i|}$ defines the performance for a single strip. But for a single strip it is then optimal to make a large detour inside the strip. This might also hold for more than one strip. So this measure can be considered to be counterintuitive.

But there are other extensions which might be interesting to consider. One could consider axis-parallel rectangles instead of full strips. Or the objects might be of arbitrary type and even not ordered by Y-coordinate. Furthermore, obstacles could intersect.

On the other hand, one might consider dynamical versions of the problem. Consider a set of axis-aligned rectangles which separately move inside the given strips in a specified direction. Compute an inspection path that takes the movement of the rectangles into account.

Finally, the question of Mark Overmars mentioned in the introduction is still open. We are searching for a roundtrip so that the maximal time interval where a point is not seen should be minimized. The main problem is whether subpolygons induced by reflex vertices have to be visited in an order along the boundary in the optimal inspection route, see also Dror et al. [7]. Note, that the corresponding subpolygons might be visited more than once in an optimal inspection route.

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