Optimal Competitive Online Ray Search with an Error-Prone Robot

Tom Kamphans and Elmar Langetepe

University of Bonn, Institute of Computer Science I
Römerstraße 164, 53117 Bonn, Germany
{kamphans,langetep}@cs.uni-bonn.de

Abstract. We consider the problem of finding a door along a wall with a blind robot that neither knows the distance to the door nor the direction towards of the door. This problem can be solved with the well-known doubling strategy yielding an optimal competitive factor of 9 with the assumption that the robot does not make any errors during its movements. We study the case that the robot's movement is erroneous. In this case the doubling strategy is no longer optimal. We present optimal competitive strategies that take the error assumption into account.

Keywords: Online algorithms, motion planning, ray search, errors.

1 Introduction

Motion planning in unknown environments is theoretically well-understood and also practically solved in many settings. During the last decade many different objectives where discussed under several robot models. For a general overview on online motion planning problems see e. g. [3, 15, 9, 17].

Theoretical correctness results and performance guarantees often suffer from idealistic assumptions so that in the worst case a correct implementation is impossible. On the other hand, practioners analyze correctness and performance mainly statistically or empirically. Therefore it is useful to investigate, how theoretic online algorithms with idealistic assumptions behave if those assumptions cannot be fulfilled. Can we incorporate assumptions of errors in sensors and motion into the analysis?

The task of finding a point on a line by a blind agent without knowing the location of the goal was considered by Gal [6, 1] and independently reconsidered by Baeza-Yates et al. [2]. Both introduced the so-called doubling strategy, which is a basic paradigma for searching algorithms, e. g., approximating the optimal search path, see [5]. Searching on the line was generalized to searching on m concurrent rays, see [8, 13, 14, 4, 7, 12].

In this paper we investigate how an error in the movement influences the correctness and the corresponding competitive factor of a strategy. The error range, denoted by a parameter δ, may be known or unknown to the strategy.

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Due to space limitations, we give only brief sketches of the proofs and refer the interested reader to [11] where we also consider a second error model.

2 The Standard Problem and the Error Model

The task is to find a point, t, on a line. Both the distance from the start position s to t, as well as the position of t (left hand or right hand to s) is unknown. A strategy can be described by a sequence $F = (f_i)_{i\in\mathbb{N}}$. f_i denotes the distance the robot walks in the i-th iteration. If i is even (odd), the robot moves f_i steps from the start to the right (left) and f_i steps back. It is assumed that the movement is correct, so after moving f_i steps away from the start point and f_i towards s, the robot has reached s. This does not hold if there are errors in the movement. In this case, every movement may be erroneous, which causes the robot to move more or less far than expected. We require that the robots error per unit is within a certain error bound, δ. Let f denote the length of a movement required by the strategy then we require that the robot moves at least $(1-\delta)f$ and at most $(1+\delta)f$ for $\delta \in [0,1[$.

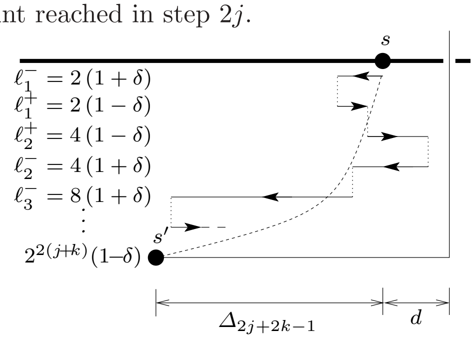
3 Finding a Point on a Line

First, we assume that the robot is not aware of making any errors. Thus, the optimal 9-competitive doubling strategy $f_i = 2^i$ [6, 2] seems to be the best choice for the robot. Let ℓ_i^+ (ℓ_i^-) be the covered distance to the right (left) in the i-th step. Now, the *drift* from s, Δ_k, is $\Delta_k = \sum_{i=1}^{k}(\ell_i^- - \ell_i^+)$.

Theorem 1. *The robot will find the door with the doubling strategy $f_i = 2^i$, if the error δ is not greater than $\frac{1}{3}$. The generated path is never longer than $8\frac{1+\delta}{1-3\delta}+1$ times the shortest path to the door.*

Proof sketch. We assume that the goal is found on the right side. For the competitivity it is the worst, if the door is hit in step $2j+2$, but located just a little bit further away than the rightmost point reached in step $2j$.

We get the worst case ratio $\frac{|\pi_{\text{onl}}|}{d} = 1 + \frac{\sum_{i=1}^{2j+1}(2\ell_i^-)}{\ell_{2j}^+ - \sum_{i=1}^{2j-1}(\ell_i^- - \ell_i^+) + \varepsilon}$ (*). This maximizes for $\ell_i^{\mp} = (1\pm\delta)2^i$, and we get $\frac{|\pi_{\text{onl}}|}{d} < 1 + 8\,\frac{1+\delta}{1-3\delta}$. We have to require $\delta \le \frac{1}{3}$, otherwise the distance $(1-3\delta)\,2^{2j}+4\delta$ from s may not exceed the point 4δ . $\square$



One might wonder if there is a strategy which takes the error δ into account and yields a smaller factor. Intuitively this seems to be impossible, but we are able to show that there is such a strategy.

Theorem 2. *In the presence of an error up to δ there is a strategy that meets every goal and achieves a competitive factor of* $1 + 8\left(\frac{1+\delta}{1-\delta}\right)^2$.

Proof sketch. We design a strategy, $F = (f_i)_{i\in\mathbb{N}}$. From (*) we conclude that it is sufficient to minimize $G_{(n,\delta)}(F) := \frac{\sum_{i=1}^{n+1} f_i}{(1-\delta)f_n - 2\delta\sum_{i=1}^{n-1} f_i}$, which is achieved by the strategy $f_i = \left(2\,\frac{1+\delta}{1-\delta}\right)^i$. This strategy is reasonable since it monotonically increases the distance to s, and we reach every goal. □

This factor is optimal. We can show that for every δ there is a strategy, F^*, that achieves the optimal factor C_δ exactly in every step, and describe F^* by a recurrence. Finally, the condition $f_i > 0$ leads to a lower bound for C_δ. Thus:

Theorem 3. *In the presence of an error up to $\delta \in [0, 1[$, there is no competitive strategy that yields a factor smaller than* $1 + 8\left(\frac{1+\delta}{1-\delta}\right)^2$.

4 Error-Prone Searching on m Rays

The robot is located at the common endpoint of m infinite rays, knowing neither the location—the ray containing t— nor the distance to t. Gal [6] showed that w.l.o.g. one can use a *periodic* and *monotone* strategy, i. e., f_i and f_{i+m} visit the same ray, and $f_i < f_{i+m}$ holds. In the error-prone setting, the start point of every iteration cannot drift away, since the start point is the only point where all rays meet.

Theorem 4. *Searching for a target located on one of m rays with an error-prone robot using a monotone and periodic strategy is competitive with an optimal factor of* $3 + 2\,\frac{1+\delta}{1-\delta}\left(\frac{m^m}{(m-1)^{m-1}} - 1\right)$ *for* $\delta < \frac{e-1}{e+1}$.

Proof sketch. It turns out that we consider the functionals $G_k(F) := \frac{\sum_{i=1}^{k+m-1} f_i}{f_k}$ in this case, which are identical to the functionals considered in the error-free m-ray search. Thus, $f_i = (m/m-1)^i$ minimizes $G_k(F)$, see [2, 6]. Ensuring monotony leads to the condition $\delta < \frac{e-1}{e+1}$. □

5 Summary

We have analyzed the standard doubling strategy in the presence of errors in movements. The robot still reaches the goal for $\delta \le \frac{1}{3}$ with a competitive ratio of $8\frac{1+\delta}{1-3\delta} + 1$. If δ is known to the strategy $f_i = \left(2\,\frac{1+\delta}{1-\delta}\right)^i$ is optimal with a competitive factor of $1 + 8\left(\frac{1+\delta}{1-\delta}\right)^2$. In the case of m rays $f_i = (m/m-1)^i$ yields $3 + 2\,\frac{1+\delta}{1-\delta}\left(\frac{m^m}{(m-1)^{m-1}} - 1\right)$ for $\delta \le \frac{e-1}{e+1} \approx 0.46$.

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